Equation Derivation and Verification Procedure

Zurek Master Equation for Quantum-to-Classical Transition (CLEAN ASCII)

Scope

This document provides an archive-safe (plain-text math) derivation outline and a concrete test/verification procedure for a unified information-thermodynamic balance law consistent with established decoherence, open quantum systems, and quantum Darwinism. The evolution law is written in a dimensionless entropy form to ensure dimensional consistency.

I. Master Evolution Law (Dimensionless Entropy Form)

Define dimensionless total entropy: Sbar_total := S_total / k_B.

Master equation (units: 1/s for all terms):

```
d(Sbar_total)/dt = Gamma_dec(H_SE) - d/dt [ I_mutual(S:E)/k_B ] + (1/(k_B*T)) *
d(DeltaF_quantum)/dt
```

Definitions (plain-text):

```
S_total = S_S + S_E (J/K)
Sbar_total = S_total / k_B (dimensionless)
I_mutual(S:E) = S(rho_S) + S(rho_E) - S(rho_SE)
S(rho) = -Tr( rho * ln(rho) )
Gamma_dec(H_SE) = (1/hbar^2) * sum_k |H_SE^(k)|^2 * tau_k (1/s)
DeltaF_quantum = F_quantum - F_classical (J)
F = - T*S (J), with S in J/K
```

II. Derivation Outline (MVP Phase I + II)

Step 1 (Open-system reduction): Start from unitary dynamics on system+environment, rho_SE(t), and define the reduced state rho_S(t) = Tr_E[rho_SE(t)]. Coarse-graining (partial trace) is the operational step that produces apparent irreversibility at the level of rho_S even when rho_SE evolves unitarily.

Step 2 (CPTP dynamics): Under standard assumptions (Born-Markov / weak coupling / secular approximation where applicable), the reduced dynamics are represented by a completely-positive trace-preserving (CPTP) semigroup, yielding the Lindblad master equation.

Lindblad form (reference model; ASCII):

```
d(rho_S)/dt = -(i/hbar) * [H_S, rho_S] + sum_k gamma_k * ( L_k*rho_S*L_k_dag -
0.5*{L_k_dag*L_k, rho_S} )
```

Step 3 (Entropy and correlations): The local entropy S_S(t) can increase because correlations with the environment grow. Mutual information I_mutual(S:E) quantifies correlations and encodes where the system's lost coherence/information went.

Step 4 (Thermodynamic driving): Define non-equilibrium free energy F(t) = - T*S. For typical decohering, dissipative couplings, DeltaF_quantum(t) decreases as superpositions relax toward effective classical mixtures (pointer statistics). The term (1/(k_B*T)) * d(DeltaF)/dt converts free-energy relaxation into an entropy-rate contribution.

Step 5 (Balance law assembly): Express entropy production in dimensionless units Sbar = S/k_B so that entropy-rate terms and decoherence-rate terms share units (1/s). The resulting balance law links: (a) decoherence rate, (b) correlation flow, and (c) free-energy relaxation.

III. Required Constraints (Kept Separate from the Scalar Law)

To avoid mixing operators with scalar rates, the following elements are imposed as constraints/conditions rather than multiplied into the scalar master equation:

- Dynamics (CPTP): rho_S(t) evolves via a Lindblad/CPTP map.
- Pointer basis (einselection): Pi_pointer = argmin_{Pi_i} Gamma_dec(Pi_i), and typically [H_SE, Pi_pointer] ~ 0.
- Objectivity (quantum Darwinism): R_redundancy >> 1, where R_redundancy := I(S:E_f)/H(S) for small environment fragments E_f.

IV. Dimensional Consistency Check

LHS: d(Sbar_total)/dt has units 1/s because Sbar_total is dimensionless.
Term 1: Gamma_dec has units 1/s by construction.
Term 2: d/dt[I_mutual/k_B] has units 1/s because I_mutual/k_B is dimensionless.
Term 3: (1/(k_B*T)) * d(DeltaF)/dt has units 1/s because (k_B*T) has units J and d(DeltaF)/dt has units J/s.

V. Verification and Test Procedure (4-Test MVP)

Test 1: CPTP / Positivity / Trace Preservation

Show either (a) explicit Lindblad form with gamma_k >= 0, or (b) Kraus representation. Verify Tr(rho_S)=1 and rho_S >= 0 for all t.

Test 2: Information Accounting (No Net Information Destruction)

Track S_S(t), S_E(t), and I_mutual(S:E). Confirm that local entropy growth is accompanied by correlation growth consistent with unitary SE dynamics.

Test 3: Pointer-State Predictability Sieve

Prepare superpositions in different bases and measure purity Tr(rho_S^2) decay rates; the basis with slowest decay is the pointer set.

Test 4: Quantum Darwinism Redundancy / Objectivity

Partition environment into fragments E_f and compute I(S:E_f) versus fragment size. Objectivity criterion: many disjoint fragments yield near-complete information about pointer states (R_redundancy >> 1).

VI. Practical Test Contexts (Non-Exhaustive)

- Cavity QED / circuit QED: controlled couplings and decoherence-rate measurement.
- Optomechanics: mesoscopic superpositions with tunable bath coupling; pointer selection via purity decay.
- Photonic scattering environments: redundancy measurements via scattered-photon fragments.
- Analog simulation: Lindblad simulation sweeps over coupling g, temperature T, and bath size N_env.

End of Document (CLEAN ASCII)